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Discriminating Rigid from Nonrigid Motion

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Contents

Abstract

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1. Introduction

Human observers report seeing three-dimensional (3D) relationships in certain changing two-dimensional (2D) images, e.g., images that represent projections of rotating solid objects (Wallach & O'Connell, 1953) or projections of rotating patterns of dots (Braunstein, 1962; Green, 1961). There has been recent interest in the minimum numbers of points and views for which subjects can make accurate judgments about 3D structure from 2D images. This interest stems in part from theoretical analyses of the minimum conditions under which an ideal observer can infer 3D structure from 2D coordinates. In this paper we relate psychophysical data to theoretical analyses for a particular judgment: discriminating rigid from nonrigid motion.[1]

Lappin, Doner, & Kottas (1980) studied the ability of subjects to judge 3D relationships on the basis of only two views. They added noise to polar projections of rotating rigid spheres by varying the number of points that were in correspondence between the views. They concluded that two views were sufficient for discriminating between different levels of noise applied to rigid structures. Braunstein, Hoffman, Shapiro, Andersen, & Bennett (1987) asked subjects to discriminate between same and different rigid structures on the basis of 2-6 views of 2-5 points. They found that human performance exceeded theoretical expectations, although some of the accuracy may have resulted from subjects exploiting the correlation that exists between 3D and 2D interpoint distances: 2D interpoint distances tend to be more similar for two projections of the same 3D object than for two projections based on different 3D objects.

Todd (1988) has provided further evidence that two views are sufficient for distinguishing between rigid and nonrigid motion. He had subjects rate the rigidity of the depicted motion for two, four or eight views of 14 connected line segments. The nonrigid displays were created by having each line segment end point rotate about an axis whose position and orientation with respect to the picture plane was selected at random. The mean ratings given by subjects for nonrigid and rigid displays were at opposite ends of a five-point rating scale. This clear discrimination between rigid and nonrigid displays did not increase with views, possibly because the effect had already reached a ceiling in the two-view condition.

In research concerned specifically with testing the applicability of Ullman's (1979) theorem to human observers, Petersik (1987) studied discrimination of rigid from nonrigid motion in displays consisting of three views of four points. This study used only rotations about a vertical axis. Nonrigid motion was produced by taking rigid displays and displacing points horizontally or vertically in the 2D projection. This method, however, does not provide a clear indication of a subject's ability to discriminate rigid from nonrigid motion. When nonrigid displays are produced by perturbing the 2D trajectories of points in a rigid display, it may be possible to

[1] Points move rigidly if all of their three-dimensional interpoint distances remain constant over time.

distinguish between rigid and nonrigid displays on the basis of the trajectories of individual points. The most obvious case is that of a parallel projection of dots rotating about a vertical axis with a perturbation inserted in the vertical direction. All of the unperturbed trajectories are horizontal lines; any perturbed trajectories in the nonrigid display may be detected merely because they deviate from horizontal lines. It is important that any task involving discriminations between rigid and nonrigid motion should require subjects to use relationships between points; the trajectories of individual points should not be discriminable between rigid and nonrigid displays.

1.1 Theoretical Developments

Theoretical investigations of structure from motion have proceeded in two directions. In the first, investigators have developed specific theorems, stating specific conditions in which image motion can be given a three-dimensional interpretation. In the second, investigators have developed a general framework, within which the specific theorems can be seen as special cases. In this section we briefly review the progress on specific theorems. A discussion of the general framework is beyond the scope of this paper, and may be found elsewhere (Bennett & Hoffman, 1988; Bennett, Hoffman, & Prakash, 1989). We then discuss the distinction between detecting and recovering rigid structures in motion, a distinction critical to the experimental work we present.

The specific theorems can be distinguished, for convenience, along three dimensions: constraints, projection, and temporal mode. Constraints are required due to the fundamental ambiguity of structure from motion, viz., any given dynamical image, no matter how rich in features or extended in time, has not just one three-dimensional interpretation but, in principle, infinitely many. A dynamic two-dimensional image does not, by itself and without further constraints, specify a unique three-dimensional interpretation; it does not because the components of motion and position along the lines of sight are lost in projection. So if, for example, one collected the data output by a video camera, it would not make sense to ask, without further constraints, what is *the* three-dimensional interpretation of that data. For this reason each theorem employs some constraint.

The constraints employed so far can be grouped into two categories: rigid and nonrigid motion. The constraint of rigid motion has been proposed by many perceptual psychologists (Gibson & Gibson, 1957; Green, 1961; Hay 1966; Johansson, 1975; Ullman, 1979; Wallach & O'Connell, 1953). The idea is that, of all possible three-dimensional interpretations of dynamic two-dimensional images, the rigid interpretations should be among the ones preferred. Kruppa (1913), building on work of Chasles (1855), first stated rigorously conditions in which image motion can be given a three-dimensional interpretation using a constraint of rigidity. Kruppa's result and others (Faugeras & Maybank, 1989; Huang & Lee, in press;

Longuet-Higgins & Prazdny, 1980; Ullman, 1979), allow arbitrary rigid motions. Other results have restricted the type of rigid motion: rotation about the vertical axis (Longuet-Higgins, 1982); rotation about an arbitrary fixed-axis (Bobick, 1986; Hoffman & Bennett, 1986; Webb & Aggarwal, 1981); rotation at a constant angular velocity (Hoffman & Bennett, 1985; 1986); and rotation in a single plane (Hoffman & Flinchbaugh, 1982). Nonrigid motion has been less studied (Bennett & Hoffman, 1985; Grzywacz & Hildreth, 1987; Koenderink & van Doorn, 1986; Ullman, 1984).

The projections employed in the theorems are two: orthographic and perspective. In orthographic projection the distance of an object from the imaging surface has no effect on its image. If one uses cartesian coordinates (x,y,z) such that coordinates x and y lie in the imaging plane and z is orthogonal, then orthographic projection is the map $(x,y,z) \mapsto (x,y)$. In perspective projection the distance of an object from the imaging surface does have an effect on its image, with greater distance leading to a smaller image. A simple model of this is given by the map $(x,y,z) \mapsto (x/z, y/z)$. Some analyses use a combination of orthographic and perspective projections, as in Ullman's (1979) "polar-parallel" projection.

The temporal modes employed are two: discrete and continuous. Discrete time analyses treat motion much like a video camera does--as a sequence of frames (Hoffman & Flinchbaugh, 1982; Huang & Lee, in press; Longuet-Higgins, 1982; Ullman, 1979). Continuous time analyses treat motion in terms of vector fields and their spatial and temporal derivatives (Hoffman, 1982; Koenderink & van Doorn, 1975, 1976, 1981; Longuet-Higgins & Prazdny, 1980; Waxman & Wohn, 1987). Again, combinations of these temporal modes are possible, though not common (Bobick, 1986).

We now consider one structure-from-motion theorem in modest detail--as an example of this line of investigation, and as an aid to understanding the distinction between detection and recovery. Bennett, Hoffman, Nicola, and Prakash (1989) prove the following result. Suppose that four points are moving in space. Suppose that one is given two distinct orthographic views of the points. And suppose that, between the two views, the four points move rigidly and are noncoplanar. Then the two views contain sufficient information to restrict the possible rigid interpretations to a one-parameter family. Moreover, if the four points do *not* move rigidly between the views then, almost surely (Lebesgue measure), the views have no possible rigid interpretation (this last statement was proved by Ullman (1977)).

We can now make clear, with the aid of this example theorem, the distinction between detection and recovery of a structure in motion. Informally, to *detect* rigid structures is to discriminate successfully between those image data (here, those two views of four points) that have rigid interpretations from those that do not; to *recover* rigid structures is to assign a rigid interpretation to each set of image data that is compatible with a rigid interpretation. Detection is necessary for recovery, but not vice versa. In our example theorem, two views of four points are sufficient to detect rigid structures, but once detection has occurred there is still an uncountable set of rigid interpretations that could be assigned. Rigidity alone, under these conditions, is an insufficient constraint to pick out one interpretation from this uncountable collection. Hence one cannot, under these conditions, recover a rigid structure.

The example theorem states that for two views of any four moving points it is theoretically possible to determine whether or not those points could lie on a rigid structure. Our first objective in the present study was to determine whether human observers can discriminate rigid from nonrigid structures at this minimum combination of points and views. A finding that human observers can make this discrimination on the basis of two views of four points would provide support for the hypothesis that human observers can exploit a rigidity constraint. Our second objective in this research was to examine the robustness of this discrimination as noise is added to a rigid display. To do this we measured the reduction in accuracy of this discrimination when points that were not part of the rigid structure were added to the display.

The first experiment was intended to demonstrate that rigid structures can be discriminated from nonrigid structures at the theoretical minimum level of two views of four points. The second experiment studied how increasing the number of views affects accuracy of this discrimination. The third experiment studied how the detection of rigid structures is affected by adding points that were not part of the rigid structure. This addresses a fundamental question: whether a rigidity constraint is likely to be useful in human vision for segregating rigid from nonrigid motion, so that 3D structure can be recovered for those points that are moving rigidly.

2. Experiment 1

The principal objective of the first experiment was to establish that subjects can discriminate rigid from nonrigid structures on the basis of two distinct views of as few as four points. The discriminability of rigid and nonrigid structures depends of course on the set of nonrigid structures that are used in the "noise" trials. As we noted earlier, the nonrigid trials must be generated in a way that does not allow discrimination between rigid and nonrigid displays on the basis of the motions of individual points. This precludes the use of random motions in the nonrigid displays and of methods in which the 2D trajectories of moving points are perturbed. Instead, both the rigid and nonrigid displays should be generated from the same individual dot motions. This suggests two methods of generating nonrigid displays, methods which can be used separately or in combination. The first assigns a different angular velocity to each point in the nonrigid displays, but has all points rotating about the same axis. The second assigns the same angular velocity to each point, but has each point rotate about a different axis. We chose the latter method because the first produces a relationship between 2D and 3D nonrigidity, viz., greater 2D nonrigidity for the displays that are nonrigid in 3D. (The measure of 2D nonrigidity that we used is described in the Stimuli section.) Producing 3D nonrigidity by varying the axis of rotation, as in the latter method, does not result in a consistent relationship between 2D and 3D nonrigidity.

2.1 Method

2.1.1 Subjects. The subjects were four undergraduate students, one graduate student, and one staff member, who were paid for their participation. Acuity of at least 20/40 (Snellen eye chart) was required in the eye used throughout the experiment. Three of the undergraduate students were run without feedback. These subjects had no knowledge of the purposes of the experiment. The remaining subjects were run with feedback. One of these subjects was the third author; the other two were generally familiar with the purposes of the experiment.

2.1.2 Design. We examined two independent variables: the number of points in a simulated object and the presence or absence of feedback. The number of points was 4, 5, or 6. Each subject responded to 80 signal trials and 80 noise trials at each of the three levels of points.

2.1.3 Stimuli. A stimulus display consisted of two views of four, five, or six light-green dots, changing position against a dark background. The two views represented a sequence of orthographic projections of points undergoing rotations in three dimensions. Initial point positions were selected at random within the volume of a sphere. The axes of rotation were determined as follows: A total of 272 points were placed at approximately equal distances on the surface of a sphere. (This was done using a three frequency dodecahedron approximation. See Pugh, 1986.) A set of potential axes of rotation was defined by connecting each of these points to the center of the sphere, with the constraint that the slant angle of each axis (relative to the viewing direction) fell within the range 45°-90°. There were 34 axes that met this constraint. For *rigid* displays, all points in the display rotated about the same axis (which was randomly selected from the set of 34 axes). For *nonrigid* displays, each point rotated about a different axis, each randomly selected without replacement from the set of 34 axes. The angle of rotation about each axis was selected from a uniform distribution over integer values between 6° and 18°.

Displays were used in the experiment only if they met three criteria: (a) nearest neighbor correspondence, (b) minimum 2D motion, and (c) minimum 3D spacing. The nearest neighbor criterion required the 2D position of each point in each view to be closer to the 2D position of that point in the other view than to the position of any other point in the other view. The minimum 2D motion criterion required that each point move between views a distance of at least 5% of the radius of the generating sphere. The minimum 3D spacing criterion required all pairs of points, in any given view, to be separated by at least 5% of the radius of the generating sphere. These three criteria were imposed to help assure (a) correct correspondence matching, (b) clearly visible motion of all points, and (c) clear separation of all points.

We developed a measure of 2D nonrigidity to determine whether the 2D projections of *nonrigid* displays were less rigid than the 2D projections of *rigid* displays. First, we computed the variance, across views, of the projected interpoint distances of each pair of points in a display. Then we computed the mean of these variances across pairs. This mean gave the measure of nonrigidity in the 2D projection. An analysis of variance (ANOVA) was conducted on the stimulus displays, using the measure of 2D nonrigidity for each randomly generated display as the dependent variable. The independent variables were 3D rigid vs. 3D nonrigid displays and number of points. The 2D nonrigidity did not differ significantly for the rigid and nonrigid displays, $F(1,79) < 1$. The main effects of number of points on 2D nonrigidity, and the interaction, were also not significant.

The SOA between views was 400 msec. In order to allow sufficient time for subjects to make a judgment, the two views were repeated until the subject responded, up to a maximum of 60 sec.

2.1.4 Apparatus. The stimuli were presented on a Hewlett-Packard Model 1321B X-Y Display with a P-31 phosphor, under the control of a PDP 11/83 computer. The maximum projected diameter of each simulated object occupied 821 plotting positions on the screen and subtended a visual angle of 2°. Points were refreshed at a rate of 17.5 Hz. The dot and background luminances at the screen were approximately 5 and 0.02 cd/m^2, respectively. Subjects viewed the displays through a tube that limited the field of view to a circular area 7.9° in diameter. A 0 [illegible] neutral-density filter was inserted in the tube to remove any apparent traces on the CRT. The eye-to-screen distance was 1.7 m.

A metal and plastic model consisting of four white spheres rigidly connected by thin black rods was used to instruct the subjects. The subjects responded by pressing one of two switches, one labeled "rigid" and the other "nonrigid." The responses (and response latencies) were recorded by the PDP 11/83.

2.2 Procedure

Each subject participated in one practice session followed by four experimental sessions. Each session began with 9 practice trials followed by a random sequence of 120 trials, consisting of 20 signal and 20 noise trials at each of the three point levels. The trials were presented in three blocks of 43 trials each. There was a 2-sec delay between each trial and a 1-min rest period between each block.

Subjects were instructed to press the "rigid" switch if the display consisted of a group of dots that was moving rigidly and to press the "nonrigid" switch otherwise. A group of dots was defined as moving rigidly if "the distance from any dot to any other dot remains the same, no matter how the group is moved." The model was used to demonstrate the rigid group condition. Subjects who were to receive feedback were told that a single tone would indicate a correct response and that two tones would indicate an incorrect response. The room was darkened 2 min before the trials began.

2.3 Results

A signal detection paradigm (Green & Swets, 1966) was used to analyze the results, with the trials containing a rigid group serving as signal trials. (We consider some of the implications of this definition of signal trials in the Discussion section.) A d' measure was computed for each subject and stimulus condition, using the proportion of "rigid group" responses on signal (3D rigid display) trials as the hit rate and the proportion of "rigid group" responses on noise (no rigid group) trials as the false alarm rate. Each d' was based on 160 trials, half of which were signal trials

The significance of the d' scores was calculated for each subject and number of points, using Marascuilo's (1970, pp. 238-240) one-signal significance test. Table 1 lists these d' values. Of a total of 18 d's (six subjects, three numbers of points) 14 were significantly different from zero ($p < .05$). For feedback subjects, 8 (of a total of 9) were significant. For nonfeedback subjects, 6 (of a total of 9) were significant. The d's for all feedback subjects and for one nonfeedback subject were significant at two views of four points. The d's for all subjects were significant at two views of five points. The mean d's for the subjects given feedback was higher than for those not given feedback (0.84 vs. 0.67) and lower for four points (0.51) than for five and six points (0.90 and 0.85), but these differences were not statistically significant.

A measure of 3D nonrigidity was developed to determine whether the amount of 3D nonrigidity in the noise displays affected the d' results. This measure was the mean across pairs of points of the variances of the 3D interpoint distances across views. (Specifically, let $\mathbf{p}_{ij} = (x_{ij}, y_{ij}, z_{ij})$ denote the position is space of point i in view j. Let $d_{ii'j}$ be the 3D distance between $\mathbf{p}_{ij}$ and $\mathbf{p}_{i'j}$. Let $\sigma^2_{ii'}$ be the variance of $d_{ii'j}$ over all views j. Then our 3D nonrigidity measure is the mean of the $\sigma^2_{ii'}$ for all distinct i and i'.) The nonrigid displays were separated into two categories--*high* and *low* 3D nonrigidity--according to whether nonrigidity was greater than or less than the median value. The proportion of false alarms was calculated separately for each category. The proportion of correct responses for the entire rigid group was used to calculate the hit rate. This provided separate measures of d' for nonrigid displays with low and high amounts of nonrigidity. Fifteen (of 18) d's were significantly different from zero when the high nonrigidity displays were used in calculating the false alarm rate and eight (of 18) were significantly different from zero when the low nonrigidity displays were used. The d' values were higher for the high nonrigidity displays than for the low nonrigidity displays in 16 of 18 comparisons (6 subjects x 3 numbers of points). The mean d's for the high nonrigidity and low nonrigidity displays were 0.99 and 0.54, respectively.

Table 1
d' Scores in Experiment 1

Subject	Number of Points		
	4	5	6
Feedback Group			
F	0.865*	1.235*	1.635*
A	0.550*	0.735*	0.280
T	0.505*	0.800*	0.925*
No Feedback Group			
G	0.345	0.715*	1.060*
L	0.475*	1.210*	0.805*
O	0.290	0.705*	0.405

*$p < .05$

These results indicate that human observers can discriminate rigid from nonrigid structures at or near the minimum level at which this discrimination is theoretically possible: two views of four points. (This is the minimum level if one assumes orthographic projection and if no constraints other than rigidity are applied.) The discriminability of rigid from nonrigid motion depends on the nonrigidity in the noise trials, as reflected in our 3D nonrigidity measure.

3. Experiment 2

Experiment 2 examines accuracy in the four-point condition as the number of views increases. Previous studies present mixed results for the effects of number of views on judgments related to recovery of 3D structure and discrimination of rigid from nonrigid motion. Doner, Lappin, and Perfetto (1984) found increased accuracy with increasing numbers of views in discriminations between different levels of spatio-temporal correlation in polar projections of rotating dot spheres. Braunstein et al. (1987) found increasing accuracy with increasing numbers of views in discriminations between same and different 3D structures. On the other hand, Todd (1988) found no increase in the discriminability of rigid from nonrigid structures as the number of views was increased beyond two. Theoretically, two views does contain sufficient information for discriminating rigid from nonrigid structures (Bennett, Hoffman, Nicola, & Prakash, 1989; Ullman, 1977), but a third view is required before a specific rigid structure can be recovered (Ullman, 1979). It is possible that human observers are more accurate in discriminating rigid from nonrigid motion when there is sufficient information to recover a specific structure. If this were the case an increase in accuracy should be expected in the three-view over the two-view condition.

Number of views, however, cannot be studied in isolation. Only two of the following three variables can be held constant as the number of views is varied: (a) rate of presentation of the views, (b) amount of rotation between views, and (c) total amount of rotation in the sequence of views. We chose to hold the first two variables constant and allow the total amount of rotation to vary with number of views. For our nonrigid displays, this resulted in an increase in our measure of 3D nonrigidity with increasing numbers of views. It is thus possible that an increase in d' with increasing views could be attributed to an increase in nonrigidity in the noise trials (suggested by Todd, personal communication, May 1, 1989). If the effect of number of views is due to the increase in 3D nonrigidity in the noise trials, we would expect that (a) d' will increase steadily with increasing numbers of views; and (b) the increase in d' will result from a decrease in the false alarm rate rather than an increase in the hit rate.

3.1 Method

3.1.1 Subjects. The subjects were four of the six subjects who had served in Experiment 1. Two subjects had received feedback in Experiment 1 and two had not.

3.1.2 Design. We examined two independent variables: number of views (2, 3, 4, 5, or 6) and SOA (66 ms or 400 ms). (Two levels were used because Todd, Akerstrom, Reichel, and Hayes, 1988, found an interaction between number of views and SOA in determining ratings of rigidity.) All displays contained four points. Each subject responded to 60 signal trials and 60 noise trials at each of the ten combinations of SOA and number of views.

3.1.3 Stimuli. The method of generating the stimuli was the same as that used in Experiment 1 with the following exceptions. The SOAs were 66 ms and 400 ms. The refresh rate for both SOAs was 15 Hz. The angles of rotation were randomly selected from a uniform distribution over integer values between 5° and 9°. For rigid displays having more than two views, a new axis of rotation was randomly

Table 2
d' Scores in Experiment 2

SOA	Subject	Number of Views				
		2	3	4	5	6
66 ms						
	F	0.905*	1.075*	1.530*	1.620*	2.030*
	A	0.200	0.390	0.460*	1.190*	1.315*
	G	0.000	0.260	1.315*	1.470*	1.575*
	L	0.490	1.630*	1.210*	1.810*	1.295*
400 ms						
	F	1.190*	1.520*	2.225*	2.300*	3.035*
	A	0.670*	0.860*	1.110*	1.165*	1.745*
	G	0.715*	1.120*	1.400*	1.045*	1.460*
	L	0.825*	1.330*	1.620*	1.420*	2.495*

* $p < .05$

selected for each additional view. For nonrigid displays having more than two views, a new axis of rotation was selected for each point in each additional view.

An ANOVA was conducted on the stimulus displays, using the 2D nonrigidity measure as the dependent variable. The independent variables were 3D rigidity, SOA, and number of views. The 2D nonrigidity was significantly different for the 3D rigid and 3D nonrigid displays, $F(1,59) = 10.8$, $p < .01$. The 2D nonrigidity measure increased significantly with number of views, $F(4,236) = 178.3$, $p < .01$. There were no other significant effects or interactions. The significant effect of 3D nonrigidity indicates that it was theoretically possible for subjects to discriminate 3D rigid from 3D nonrigid displays on the basis of 2D nonrigidity. This seems unlikely, however, as the variance in the 2D nonrigidity measure accounted for by 3D nonrigidity was 0.3%, compared to 38.2% accounted for by number of views. The means of the 2D nonrigidity measures were .0053 for the 3D rigid displays and .0058 for the 3D nonrigid displays. The means for the displays with 2-6 views were .0012, .0029, .0053, .0077, and .0107, respectively. The units are squared distances in a unit sphere.

3.2 Procedure

Each subject participated in one practice session followed by 10 experimental sessions. Each session began with 9 practice trials followed by a random sequence of 120 trials, consisting of 12 signal and 12 noise trials at each of the 5 view levels. The trials were presented in three blocks of 43 trials each. Half the experimental sessions were at the short SOA, the other half at the long SOA. The order of SOA was alternated between sessions with half the subjects beginning with the long SOA and the other half beginning with the short SOA. The procedure was otherwise the same as in Experiment 1.

3.3 Results

A d' was computed for each subject and stimulus condition (Table 2). For the short SOA 15 of the 20 d's were significantly different from zero, $p < 0.05$. Of the five that were not significant, three were at the 2 view level and two were at the 3 view level. For the long SOA all 20 d's were significantly different from zero ($p < 0.05$).

A two-way ANOVA was conducted with SOA and number of views as the independent variables. There were two significant effects. The main effect of SOA, $F(1,3) = 16.83$, $p < 0.05$, $\omega^2 = 0.08$, showed an increase in d' with longer SOA (1.46 vs. 1.09). The main effect of number of views, $F(4,12) = 29.16$, $p < 0.01$, $\omega^2 = 0.44$, showed an increase in d' with greater numbers of views. Post hoc comparisons (Tukey's HSD test) showed significant differences for 2 views vs. 3, 4, 5, and 6 views; 3 views vs. 5 and 6 views; and 4 views vs. 6 views.

As in Experiment 1, d's were calculated with the nonrigid displays divided into high and low 3D nonrigidity subgroups. For the high nonrigidity displays 36 of 40 d's were significantly different from zero with a mean d' of 1.50. For the low nonrigidity displays 29 of 40 were significantly different from zero with a mean d' of 1.07. The d' values were greater for the high nonrigidity displays than for the low nonrigidity displays in 37 of 40 comparisons (4 subjects x 2 SOA's x 5 numbers of views).

The relationship between number of views and 3D nonrigidity, d', hit rate, and false alarm rate is shown in Figure 1. The 3D nonrigidity measure increased with number of views. There was a corresponding decrease in the false alarm rate. The hit rate remained constant, indicating that the increase in d' was due to a decrease in the false alarm rate. This is the pattern of results that would be expected if the effect of number of views was due to the increase in the 3D nonrigidity that occurred with increasing numbers of views. This provides a further indication of subjects' sensitivity to variations in 3D nonrigidity and confirms the usefulness of the 3D nonrigidity measure as a predictor of performance in discriminating rigid from nonrigid motion.

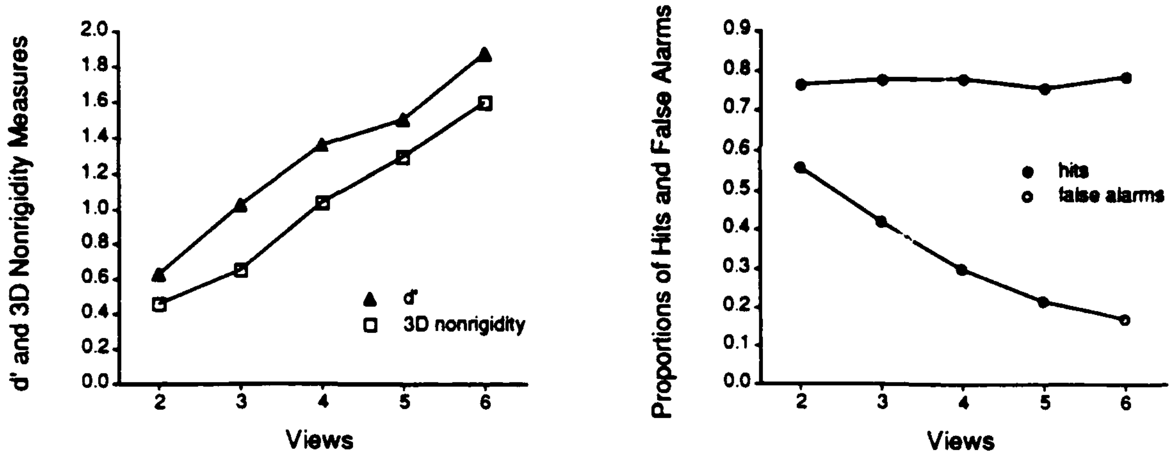

Figure 1. d', 3D nonrigidity, proportion of hits, and proportion of false alarms as functions of the number of views (Experiment 2). (In order to use the same abscissa values for d' and the 3D nonrigidity measure, the nonrigidity measure is multiplied by 1000 in this figure and in Figures 2 and 3.)

4. Experiment 3

Two orthographic views of four points are theoretically sufficient to determine whether or not a 3D motion is rigid (Bennett, Hoffman, Nicola, & Prakash, 1989; Ullman, 1977), and the results of Experiments 1 and 2 indicate that subjects can make this discrimination at these minimum levels of points and views. For displays containing more than four points, the same theoretical analysis can be used to determine whether a display contains any subset of four points that is moving rigidly. It is important to know whether subjects can also determine whether rigid motion is present under these conditions; the usefulness of a rigidity constraint would be severely limited if such a constraint could be applied only when all moving elements were part of the same rigid structure. Experiment 3 included displays in which four points were moving rigidly but which, in addition, had from one to four points that were not part of the rigid structure. The subject's task, rather than indicating whether the observed structure was rigid or nonrigid as in Experiments 1 and 2, was to determine whether the display contained at least four points that moved together rigidly.

4.1 Method

4.1.1 Subjects. The subjects were three of the four subjects from Experiment 2 and one graduate student who had not served in Experiments 1 or 2. Three of the subjects were naive as to the purposes of the experiment; one subject was the third author. As a precondition for participating in this experiment, each subject was required to achieve a d' of 1.2 or better in a screening session in which they responded to 100 trials of 12 views of 4 points. This criterion assured that subjects were performing, on trials with no noise points, at a level comparable to performance in Experiment 2. One of the four subjects failed to meet the criterion in the first screening session but succeeded in doing so in a second screening session.

4.1.2 Design. We examined two independent variables: number of views (2, 3, 4, or 12) and number of noise points (0, 1, 2, 3, or 4). Each subject responded to 60 signal trials and 60 noise trials at each of the 20 combinations of number of views and noise points.

4.1.3 Stimuli. The method of generating the stimuli was the same as that used in Experiments 1 and 2, with the following exceptions: The 2D minimum motion criteria for a display had to be met for each point for at least one transition between views rather than for all transitions. This change was made because of difficulty in generating 12-view displays that satisfied the more stringent criterion. Also, there was a change of two parameters: SOA and range of rotation angle for transitions. Two SOAs were used, 80 ms and 240 ms. (These were selected on the basis of Todd's observations, personal communication November 1988, and our own observations of the SOAs required for perception of smooth motion for two-view and multiple-view displays.) The refresh rate for both SOAs was 12.5 Hz. The long SOAs were used for the two view displays and the short SOA for the 3, 4 and 12 view displays. The angles of rotation were randomly selected from a uniform distribution of integer values between 5° and 7°. The larger rotation angles used in the previous experiments were eliminated because they appeared to interfere with the perception of smooth motion at the 80-ms SOA.

An ANOVA was conducted on the 2D nonrigidity measure, with 3D rigidity, number of views, and number of noise points as the independent variables. The only significant effect was the main effect of number of views, $F(3,177) = 1510.0$, $p < .01$. The means for 2, 3, 4, and 12 views were 0.0009, 0.0019, 0.0029, and 0.0133.

4.2 Procedure

Each subject participated in one or more screening sessions (described above), one practice session, and 24 experimental sessions. Each experimental session began with 5 practice trials followed by a random sequence of 100 trials, consisting of 10 signal and 10 noise trials at each of the 5 noise point levels. The trials were presented in three blocks of 35 trials each. There were 6 sessions at each of the 4 levels of number of views. The number of views across the 24 sessions was in the order 12, 4, 3, 2, 2, 3, 4, and 12 views, repeated three times.

As in Experiment 1 there was a 2 sec delay between each trial and a 1 min rest period between each block. The subjects were instructed to press the "rigid" switch if the display contained a group of dots that was moved together rigidly and to press the "nonrigid" switch otherwise. A group of dots was defined as moving together rigidly if "at least four dots maintain constant distances from each other regardless of how the entire group moves."

4.3 Results

A d' was computed for each subject and stimulus condition (Table 3). Of 80 d's, 48 were significantly different from zero, $p < 0.05$. For 0 noise points 15 (of 16) d's were significantly different from zero. For 4 noise points 7 (of 16) d's were significantly different from zero.

The independent variables in the ANOVA were number of noise points and number of views. There were two significant effects. The main effect of number of noise points, $F(4,12) = 26.79, p < 0.01, \omega^2 = 0.34$, showed a decrease in d' with more noise points. The mean d' values for 0, 1, 2, 3, and 4 noise points were 0.97, 0.54, 0.45, 0.37, and 0.40, respectively. Post hoc comparisons showed only the differences between 0 noise points and nonzero noise point conditions to be significant. The main effect of number of views, $F(3,9) = 10.43, p < 0.01, \omega^2 = 0.21$, showed an increase in d' with greater numbers of views. The mean d's for 2, 3, 4, and 12 views were 0.34, 0.52, 0.49, and 0.84, respectively. Post hocs comparisons showed only the differences between 12 views and smaller numbers of views to be significant.

In the previous experiments we examined the relationship between accuracy of discrimination and a measure of 3D nonrigidity for the noise trials. For those experiments the 3D nonrigidity for the signal trials was always zero. In Experiment 3, 3D nonrigidity increased for the signal trials as additional noise points were added. It is likely that discriminability in this experiment was based on a relationship between 3D nonrigidity in the signal trials and 3D nonrigidity in the noise trials. We examined two obvious relationships: the ratio of the nonrigidity measure (signal trials/noise trials) and the difference in the measure (noise trials - signal trials). The correlations with d', across the 20 combinations of views and noise points, were -.65 for the ratio measure and .87 for the difference measure. We therefore present the difference measure in Figures 2 and 3. Figure 2 shows the effects of number of noise points on d' and on the difference between noise and signal trials in 3D nonrigidity. The hit rate and false alarm rate are also shown. Figure 3 presents these effects as the number of views increases from 2 to 12. These results suggest that the difference in nonrigidity, or some related quantity, accounts both for the effects of points and for the effects of views. These effects are due primarily to changes in the false alarm rate.

Table 3
d' Scores in Experiment 3

Number of Views	Subject	Number of Noise Points				
		0	1	2	3	4
2	F	0.740*	0.420	0.170	0.125	0.645*
	M	0.300	0.545*	0.000	-0.135	0.000
	G	0.695*	0.320	0.895*	0.000	0.105
	L	0.740*	0.380	0.555*	-0.045	0.305
3	F	1.200*	0.815*	0.725*	0.160	0.320
	M	0.630*	0.505*	0.245	0.175	0.090
	G	0.550*	0.445	0.730*	0.490*	0.305
	L	1.045*	0.515*	0.310	0.385	0.715*
4	F	0.950*	0.595*	0.465*	0.415	0.375
	M	1.040*	0.505*	0.260	0.565*	0.530*
	G	0.505*	0.375	-0.205	0.650*	0.510*
	L	0.940*	0.595*	0.275	0.180	0.220
12	F	1.560*	0.850*	0.695*	0.945*	0.815*
	M	2.005*	0.865*	0.870*	0.480*	0.660*
	G	1.345*	0.465*	0.685*	0.660*	0.250
	L	1.330*	0.375	0.550*	0.800*	0.555*

*$p < .05$

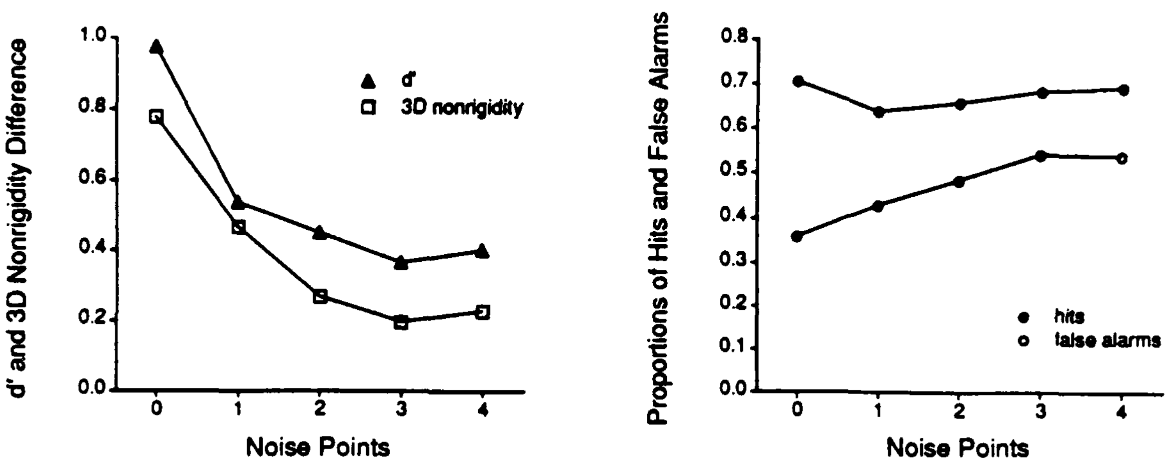

Figure 2. d', difference in 3D nonrigidity (noise nonrigidity - signal nonrigidity), proportion of hits, and proportion of false alarms as functions of the number of noise points in the signal displays (Experiment 3).

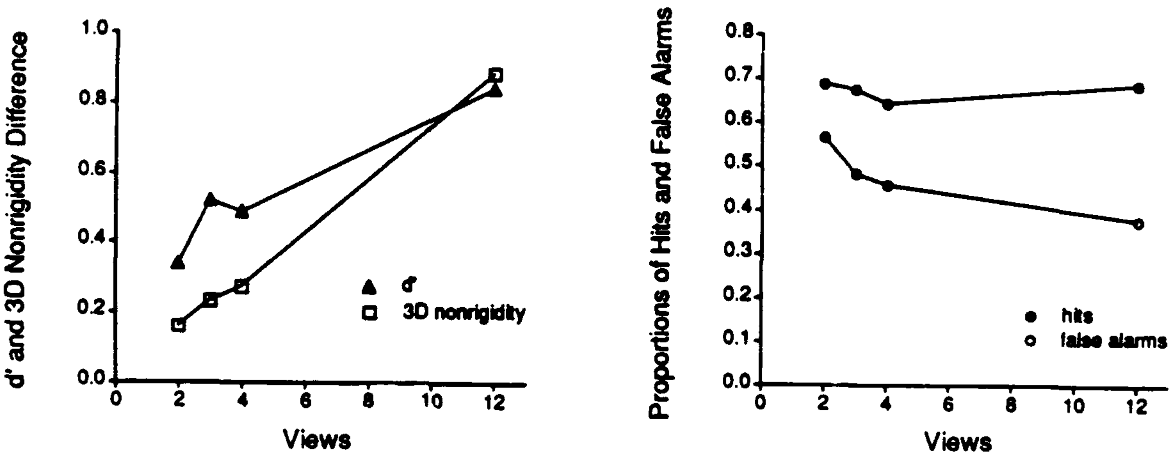

Figure 3. d', difference in 3D nonrigidity (noise nonrigidity - signal nonrigidity), proportion of hits, and proportion of false alarms as functions of the number of views (Experiment 3).

5. Discussion

Human observers can discriminate rigid from nonrigid motion at the minimum level of points and views at which such discrimination is theoretically possible on the basis of a rigidity constraint alone: two views of four points. For discriminations between displays in which all points were either moving rigidly or were rotating about separate axes, accuracy depended on the deviation of the nonrigid displays from rigid motion. Our measure of this deviation, the mean across pairs of points of the variance in the interpoint distance over views, was related to the discriminability of rigid from nonrigid displays. This measure is based on the 3D structure used to generate the displays. The usefulness of this measure is especially interesting in the case of the two-view displays, because the same two view displays can be generated from an infinite number of rigid 3D structures (Bennett, Hoffman, Nicola, & Prakash, 1989).

Increasing the number of points in a rigidly moving group does not lead to a clear increase in accuracy, although there was a nonsignificant increase from four to more than four points. It is certainly possible that an effect of points would be found for larger numbers of points--numbers sufficient to give the configuration a clear shape. Increasing the number of views did increase accuracy of discrimination, but this can be attributed to the increase in nonrigidity of the nonrigid displays. With points rotating about separate axes, the variance of the distances between pairs of points increases with number of views. Our measure of 3D nonrigidity, based on these variances, correlated .985 with d' across the five levels of views.

Although human subjects can discriminate rigid from nonrigid structures at the minimum level of points and views at which this discrimination is theoretically possible, accuracy drops sharply when even one point that is not part of the rigid structure is added to a rigid display. It appears that human observers are not proficient at analyses that require testing subgroups of points to determine whether one subgroup is present that is moving rigidly. (With five points there would be five such subgroups to test. This may not seem to be much of a processing load from a computational viewpoint, but five subgroups involving six distances each in one display may be difficult for human subjects to process.) These results may appear to be in conflict with Ullman's (1979) well known demonstration that two concentric cylinders differing in diameter are easily segregated by the human visual system. This demonstration, however, is not directly comparable to the present stimuli. The demonstration used a large number of points and views rather than the minimal numbers used in the present research. Perhaps more importantly, the motion in the demonstration was rotation about a fixed axis at a constant angular velocity. Bennett and Hoffman (1985) have shown that a fixed-axis constraint is sufficient mathematically for recovering 3D structure from four orthographic views of two points or three orthographic views of four points; a rigidity constraint is not necessary. Demonstrations by Braunstein (1983) and Ramachandran, Cobb, &

Rogers-Ramachandran (1988) also indicate that the perceptual segmentation of two rotating cylinders may not be based entirely on the use of a rigidity constraint.

The sharp drop in accuracy in detecting the presence of a rigid structure when noise points were added to the structure is consistent with Lappin et al.'s (1980) results with larger numbers of dots. In that study accuracy in determining which of two displays had more coherent motion was highest when one of the displays was completely rigid, but dropped sharply when both displays contained nonrigid motion. If the subjects in the present experiments were primarily engaged in detecting nonrigid motion, rather than detecting rigid groups of points, it is not surprising that accuracy would drop sharply when both the signal trials and noise trials included nonrigid motion.

Discrimination between rigid and nonrigid structures, at least on the basis of small numbers of points and views, does not appear to be an easy task for human subjects. Subjective reports indicate that this task requires careful attention. It is possible that the task could be performed with less effort if the nonrigid motions differed even more from the rigid motions. In our displays, the same center of rotation was used for all points whether or not they were part of a rigid structure. Generically, feature points that are moving independently would probably not have the same center of rotation. This probably made discriminations especially difficult in the present study, but this was necessary to prevent a consistent relationship between nonrigidity in the 2D projection and nonrigidity in 3D.

In presenting a signal detection analysis of the present experiments we chose to define displays containing groups of at least four points moving together rigidly as signal displays, and displays lacking such rigid groups as noise displays. Our results suggest that the opposite interpretation may be worth considering. Discrimination of rigid from nonrigid motion may be conceived of as detecting deviations from constant interpoint distances in 3D, that is, detecting nonrigidity. Thus in Experiments 1 and 2 the rigid displays might have been defined as the "noise displays" and the nonrigid displays as the "signal plus noise displays." Increasing the 3D nonrigidity of the nonrigid displays by increasing the number of views in Experiment 2 could then be described as increasing the signal strength, with the expected result of increasing d'. In Experiment 3 subjects may have been discriminating between levels of nonrigidity (i.e., between two levels of signal) rather than detecting rigid groups. Introspective reports suggest that subjects were both looking for rigid groups and looking for deviations from rigidity. The relationship between signal detection concepts and the discrimination of rigid from nonrigid motion would be worth exploring further with additional experimental manipulations.

In conclusion, these experiments reveal that human subjects are surprisingly good at some aspects of analyzing 3D structures and surprisingly poor at others. Human subjects can discriminate rigid from nonrigid motion at exactly the minimum levels of points and views specified by theoretical analyses, suggesting that such analyses may be of relevance to the study of human vision. But when the task is changed to determining whether a rigid structure is present in noise, performance falls off sharply with even one noise point. We need to look further into the issue of whether a rigidity constraint is useful in perceptual grouping, or whether other constraints must determine grouping before a rigidity constraint can be applied.

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